Introduction to Algebra

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Problem Set 1

In every homework set there will be a certain number of ♣ assigned to each problem. Complete at least 25♣ in this homework set. For complete mastery get 40♣ or more.

Problem 1.1 (1♣) Compute each of the following:

(a) $3^2 + 4 \times 2$ (b) $8/(6-2)+5$ (c) $(5-8) \times (2+7)$

(d) $8^2/4^2 + 3 \times 4$ (e) $(3^3 - 5^2) \times 5 - 8$ (f) $11 \times 6^{(2^2-3)}$

(g) $(20)(19) \div (20)(20)$

Problem 1.2 (2♣) Compute each of the following:

(a) $63 \times \frac{2}{7} \times \frac{2}{63}$ (b) $\frac{1}{4} \times 48 \times 97 \times \frac{1}{12}$ (c) $273 - 8198 - 274 + 8200$

Problem 1.3 (2♣) Simplify the following fractions:

(a) $\frac{4\times6\times7}{7\times4}$ (b) $\frac{3\times8}{27}$

Problem 1.4 (3♣) Evaluate the following expression by factoring the numerator first:

$$\frac{99+88-77+66}{11}$$

Problem 1.5 (3♣) Factor the expression $2r^2(r^2+1) - 8r(r^2+1)$ as completely as you can.

Problem 1.6 (3♣) Simplify the following fraction:

$$\frac{\frac{3x}{4x-4}}{\frac{9x^2}{x-1}}$$

Problem 1.7 (5♣) Evaluate the following expressions without straightforward multiplication:

(a)7×88 (b)12×399 (c)23×1997

Problem 1.8 (5♣) Factor the expression $2r(r-7) + 8r - 56$

Problem 1.9 (5♣) Write the expression $2 + \frac{4}{2z-1} - \frac{3}{z} + \frac{z}{2z^2-z}$ as a single fraction.

Problem 1.10 (8♣) Alice, Bob and Carl each think of an expression that is a fraction with 1 in the numerator and a constant integer times some power of x in the denominator. The simplest common denominator of Alice's and Bob's expression is $4x^2$. The simplest common denominator of Bob and Carl's expressions is $12x^3$. The simplest common denominator of Alice and Carl's expressions is $6x^3$. Find all possible expressions that could be Carl's expression.

Problem 1.11 (8♣) Factor the expression $x^2 + 5x + 4$ by finding the numbers that correctly fill the blanks below:

$$(x + \_)(x + \_)$$

Note that the numbers that go in the blank are not the same.

Writing Problems

All of the following problems have a little bit of writing to do. For each problem try to write 2-3 sentences at **minimum**

Problem 1 We saw in class that $5 - 2 \neq 2 - 5$ because the associative property does not apply to subtraction. Is it ever possible to switch the orders of the numbers *without* changing the value of the difference

Problem 2

(a) Richard expanded the product of $(-2) \times (5 - 3)$ like this:

$$(-2) \times (5 - 3) = (-2) \times 5 + (-2) \times (3) = -10 + (-6) = -16$$

Where did he go wrong? Give the correct method for the solution.
(b) Stanley subtracted the equation $3 = 4 - 1$ from the equation $16 = 2 + 4 \times 3 + 2$ and created the equation

$$16 - 3 = 2 + 4 \times 3 + 2 - 4 - 1$$

is this new equation true? If not where did Stanley go wrong?

Problem 3 Is the following correct:

$$\frac{5 + 3x}{x} = \frac{5 + 3\cancel{x}}{\cancel{x}} = 5 + 3 = 8?$$

If not, explain why it is not correct.

Problem 4 MathWizard likes to play a fun number trick on his friends. She tells them to think of a number. She then tells them to subtract a number from 7 and multiply the result by 3. To this product she tells them to add half the difference when 36 is subtracted from 8 times their number. How can Math Wizard use these steps to quickly figure out what his friends' starting number is?

Professional Problem

In this section we will be discovering new mathematical ideas with the knowledge we have found so far. Make sure in your homework to start on a new page for this section.

(a) Find the sum of $1 + 2 + 3$

(b) Find the sum of $1 + 2 + 3 + 4$

(c) Find the sum of $1 + 2 + 3 + 4 + 5$

(d) Compare your answers for the first three parts to 3×4, 4×5, and 5×6, respectively. Use your observation to guess what $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$ is, then add the 10 numbers and see if you are right

(e) Guess an expression in terms of n that is always equal to $1 + 2 + 3 + \cdots + (n - 1) + n$ no matter what the positive integer n is.

(f) Add $n + 1$ to your expression from part (e). Find a common denominator, add the fractions, then factor the numerator as much as possible. Does the result confirm your guess from part (e)?

(g) With our knowledge simplify the expression

$$\frac{1 \cdot 2 \cdot 3 \cdot ... \cdot (n-1) \cdot n}{1 + 2 + 3 + ... + (n-1) + n}$$

(Modified from 2017 AMC 8 Problem 5)